General instructions for Students: Whatever be the notes provided, everything must be copied in the Mathematics copy and then do the HOMEWORK in the same copy.

23. If $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$, prove that

$a^{q-r} \bullet b^{r-p} \bullet c^{p-q} = 1$

Solution: $a^{q-r} \bullet b^{r-p} \bullet c^{p-q}$

$= (xy^{p-1})^{q-r} \bullet (xy^{q-1})^{r-p} \bullet (xy^{r-1})^{p-q}$

$= x^{q-r} \bullet x^{r-p} \bullet x^{p-q} \bullet (y^{p-1})^{q-r} \bullet (y^{q-1})^{r-p} \bullet (y^{r-1})^{p-q}$

$= x^{q-r+r-p+p-q} \bullet y^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$

$= x^0\, y^0 = 1$ **Ans.**

28. If $x = 10^3 \times 0.0099$, $y = 10^{-2} \times 110$, find the value of $\sqrt{\frac{x}{y}}$

Solution: $x = 10^3 \times 0.0099 \implies x = 9.9$

$y = 10^{-2} \times 110 \implies y = 1.1$

$\sqrt{\frac{x}{y}} = \sqrt{\frac{9.9}{1.1}} = \sqrt{\frac{99}{11}} = \sqrt{9} = 3$ **Ans.**

33. If $\left(\frac{p^{-1}q^2}{p^2q^{-4}}\right)^7 \div \left(\frac{p^3q^{-5}}{p^{-2}q^3}\right)^{-5} = p^xq^y$, find $x + y$,

where p and q are different positive primes.

Solution: $\left(\frac{p^{-1}q^2}{p^2q^{-4}}\right)^7 \div \left(\frac{p^3q^{-5}}{p^{-2}q^3}\right)^{-5} = p^xq^y$

$\implies (p^{-1-2}q^{2+4})^7 \div (p^{3+2}q^{-5-3})^{-5} = p^xq^y$

$\implies (p^{-3}q^6)^7 \div (p^5q^{-8})^{-5} = p^xq^y$

$\implies \frac{p^{-21}q^{42}}{p^{-25}q^{40}} = p^xq^y$

$\implies p^{-21+25}\, q^{42-40} = p^x\, q^y$

$$\Rightarrow \quad p^4 q^2 = p^x q^y$$

$$\therefore \quad x = 4 \text{ and } y = 2$$

Now, $x + y = 4 + 2 = 6$ **Ans.**

34. (iv) Solve the equation for x : $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$

Solution: $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$

$$\Rightarrow \left\{(2^2)^{\frac{1}{3}}\right\}^{2x+\frac{1}{2}} = 2^{-5}$$

$$\Rightarrow \left(2^{\frac{2}{3}}\right)^{2x+\frac{1}{2}} = 2^{-5}$$

$$\Rightarrow 2^{\frac{4x}{3}+\frac{1}{3}} = 2^{-5}$$

$$\Rightarrow 2^{\frac{4x+1}{3}} = 2^{-5}$$

$$\therefore \frac{4x+1}{3} = -5 \Rightarrow 4x + 1 = -15$$

$$\Rightarrow 4x = -15 - 1$$

$$\Rightarrow 4x = -16$$

$$\Rightarrow x = -4$$ **Ans.**

37. If $\frac{9^n\, 3^2\, 3^n - (27)^n}{3^{3m}\, 2^3} = \frac{1}{27}$ **, prove that** $m = 1 + n$

Solution: $\frac{9^n\, 3^2\, 3^n - (27)^n}{3^{3m}\, 2^3} = \frac{1}{27}$

$$\Rightarrow \frac{(3^2)^n\, 3^2\, 3^n - (3^3)^n}{3^{3m}\, 2^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{2n}\, 3^2\, 3^n - 3^{3n}}{3^{3m}\, 2^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{2n+n}\, 3^2 - 3^{3n}}{3^{3m}\, 2^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{3n}\, 3^2 - 3^{3n}}{3^{3m}\, 2^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{3n}(3^2-1)}{3^{3m}\,2^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{3n-3m}(9-1)}{8} = 3^{-3}$$

$$\Rightarrow \frac{3^{3n-3m}\,8}{8} = 3^{-3}$$

$$\Rightarrow 3^{3(n-m)} = 3^{-3}$$

$$\therefore 3(n-m) = -3$$

$$\Rightarrow n - m = -1$$

$$\Rightarrow m = 1 + n$$ **Proved.**

40(i) Solve the equation: $3(2^x + 1) - 2^{x+2} + 5 = 0$

Solution: $3(2^x + 1) - 2^{x+2} + 5 = 0$

$$\Rightarrow 3(2^x + 1) - 2^x \bullet 2^2 + 5 = 0$$

$$\Rightarrow 3(p + 1) - p \bullet 2^2 + 5 = 0 \qquad \{2^x = p \ \ (\text{Say})\}$$

$$\Rightarrow 3p + 3 - 4p + 5 = 0$$

$$\Rightarrow -p = -8$$

$$\Rightarrow p = 8$$

Now, $2^x = p$

$$\Rightarrow 2^x = 8 \qquad \{p = 8\}$$

$$\Rightarrow 2^x = 2^3$$

$$\therefore x = 3$$ **Ans.**

Chapter test

3. If $p = x^{m+n} \bullet y^l$, $q = x^{n+l} \bullet y^m$ **and** $r = x^{l+m} \bullet y^n$, **prove that**

$$p^{m-n} \bullet q^{n-l} \bullet r^{l-m} = 1$$

Solution: LHS $\Rightarrow p^{m-n} \bullet q^{n-l} \bullet r^{l-m}$

$$= (x^{m+n} \bullet y^l)^{m-n} \bullet (x^{n+l} \bullet y^m)^{n-l} \bullet (x^{l+m} \bullet y^n)^{l-m}$$

$$= x^{m^2-n^2} \bullet x^{n^2-l^2} \bullet x^{l^2-m^2} \bullet y^{l(m-n)} \bullet y^{m(n-l)} \bullet y^{n(l-m)}$$

$$= x^{m^2-n^2+n^2-l^2+l^2-m^2} \bullet y^{lm-nl+mn-ml+nl-mn}$$

$$= x^0 \bullet y^0 = 1 \quad \text{RHS} \qquad \text{Proved.}$$

8. If $3^x = 5^y = (75)^z$, show that $z = \frac{xy}{2x+y}$

Solution: $3^x = 5^y = (75)^z = k$ (say)

$3 = k^{\frac{1}{x}} \qquad 5 = k^{\frac{1}{y}}$

Now, $\quad (75)^z = (3 \times 5^2)^z$

$$\Rightarrow \quad k = \left\{k^{\frac{1}{x}} \times \left(k^{\frac{1}{y}}\right)^2\right\}^z$$

$$\Rightarrow \quad k = k^{\frac{z}{x}} \times k^{\frac{2z}{y}}$$

$$\Rightarrow \quad k = k^{\frac{z}{x}+\frac{2z}{y}}$$

$$\Rightarrow \quad k^1 = k^{\frac{yz+2zx}{xy}}$$

$$\therefore \quad \frac{yz+2zx}{xy} = 1$$

$$\Rightarrow \quad \frac{z(y+2x)}{xy} = 1$$

$$\Rightarrow \quad z = \frac{xy}{2x+y} \qquad \text{Proved.}$$

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HOMEWORK

EXERCISE – 8

QUESTION NUMBERS: 22, 27, 31, 34(i), (iii); 35(ii) and 39

CHAPTER TEST: 6(i), (ii); 7, and 9(iii), (iv)

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